Searching for physics beyond the Standard Model through the dipole interaction

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Abstract. The magnetic dipole interaction played a central role in the development of QED, and continued in that role for the Standard Model. The muon anomalous magnetic moment has served as a benchmark for models of new physics, and the present experimental value is larger than the standard-model value by more than three standard deviations. The electric dipole moment (EDM) violates parity (P) and time-reversal (T) symmetries, and in the context of the CPT theorem, the combination of charge conjugation and parity (CP). Since a new source of CP violation outside of that observed in the K and B meson systems is needed to help explain the baryon asymmetry of the universe, searches for EDMs are being carried out worldwide on a number of systems. The standard-model value of the EDM is immeasurably small, so any evidence for an EDM would signify the observation of new physics. Unique opportunities exist for EDM searches using polarized proton, deuteron or muon beams in storage rings. This talk will provide an overview of the theory of dipole moments, and the relevant experiments. The connection to the transition dipole moment that could produce lepton flavor violating interactions such as $\mu^+ \to e^+\gamma$ is also mentioned.

1. Introduction

Measurements of dipole moments have played an important role in our understanding of the subatomic world. Contrary what we teach our undergraduates in modern physics, the Stern-Gerlach experiment did not motivate the invention of spin, however the modern interpretation of their result is that the magnetic moment of the electron is one Bohr-magneton [3] with a g-value of two. While "a spinning electron" was proposed by Compton to explain ferromagnetism [1], it was the introduction of spin and the associated magnetic moment by Uhlenbeck and Goudsmit to explain the fine-structure in atomic spectra [2] that was the beginning of spin physics as we now know it [3].

The discovery of anomalous magnetic moments was a critical event for twentieth-century physics that began with the observations that the hyperfine structure of hydrogen (HHFS) was too large [4] when compared to the standard (Dirac) theory [5]. The Dirac equation[1]

$$i\left(\partial_\mu - ieA_\mu\left(x\right)\right)\gamma^\mu\psi\left(x\right) = m_e\psi\left(x\right),\tag{1}$$

predicted a magnetic dipole moment (MDM) for the electron

$$\vec{\mu}_e = g_e\left(\frac{Qe}{2m_e}\right)\vec{s}, \text{ where the anomaly, defined as } a_e = \frac{(g_e - 2)}{2}, \text{ is zero}\tag{2}$$

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[1] Throughout this paper I adopt the convention $e > 0$.

since in Dirac theory $g_e \equiv 2$. The increase in the hydrogen hyperfine levels could be interpreted as coming from an additional magnetic moment. Motivated by the HHFS dilemma, Schwinger [6] carried out what today is called "the first loop calculation", and predicted that the electron had an additional (anomalous) magnetic moment, $a_e = \alpha/2\pi$. The concurrent precision spectroscopy measurements of Kusch and Foley [7], obtained a value for g_e that was in good agreement with Schwinger's prediction.

It took some years before the equivalent measurement was made for the muon. The spin-rotation experiment of Garwin, et al. [8], which was one of the pioneering experiments [8, 9] that observed parity violation in muon decay, found that the observed rate of spin rotation gave $g_\mu = 2.0 \pm 0.10$. This result indicated "the very strong probability that the spin of the μ^+ is $\frac{1}{2}$," thereby providing the first indication that the muon behaved like a heavy electron. A second muon spin rotation experiment by Garwin *et al.* [10], obtained a 12% measurement of the muon anomaly, $a_\mu^+ = 0.001\,13^{+0.000\,16}_{-0.000\,12}$ which agreed very well with the expected Schwinger value of $\alpha/2\pi \simeq 0.001\,161\ldots$. This experiment showed conclusively that the muon did indeed have the characteristics of a heavy electron.

In 1950, well in advance of the famous Lee-Yang paper [11], Purcell and Ramsey [12] observed that there was no evidence for parity conservation in the nuclear force and that an electric dipole moment (EDM) of the neutron would violate parity invariance P. This was of course the correct New-Physics effect to look for, but in the wrong place. Their initial experiment [13], which went unpublished until after the discovery of parity violation, achieved a limit of $|d_n| < 5 \times 10^{-20}\ e \cdot \text{cm}$, a null result which has been pushed down to $2.9 \times 10^{-26}\ e \cdot \text{cm}$ during the subsequent fifty-some years. It was realized in 1957 [14, 15] that an EDM would also violate time-reversal symmetry, T, and by implication CP. This can be seen by considering the Hamiltonian for dipole interactions: $H = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E}$. The magnetic moment transforms like a spin (a pseudovector) as does the EDM. The electric field is a vector while the magnetic field is a pseudovector. The combination $\vec{\mu} \cdot \vec{B}$ is even under C, P and T. The EDM term, $\vec{d} \cdot \vec{E}$, is odd under P and T, thus they are not good symmetries of this Hamiltonian. Presumably, new, as yet undiscovered sources of CP violation are responsible for the matter-antimatter asymmetry in the universe, and would partially explain why we are here.

2. The Dipole Operators

As mentioned above, the Dirac equation is inadequate to describe the measured magnetic moment of the electron. It is necessary to add a "Pauli" term

$$\frac{Qe}{4m_e} a_e F_{\mu\nu}(x) \sigma^{\mu\nu} \psi(x) \tag{3}$$

which in modern language is a dimension 5 operator that must arise from loops in a renormalizable theory. New Physics (NP) can also contribute through loops, with $a(\text{NP}) = C(m/\Lambda)^2$ where $C \simeq \mathcal{O}(1)$, or $\simeq \mathcal{O}(\alpha)$ in weak coupling loop scenarios. In the same spirit, one could add the following Pauli-like term

$$\frac{i}{2} d_e F_{\mu\nu}(x) \sigma^{\mu\nu} \gamma_5 \psi(x) \quad \text{with} \quad \vec{d} = \eta \left(\frac{Qe}{2mc} \right) \vec{s}, \tag{4}$$

which represents the electric dipole moment interaction. The quantity η plays the role for the EDM that g plays for the MDM.

The electromagnetic current is $\left\langle f(p') \left| J_\mu^{em} \right| f(p) \right\rangle = \bar{u}_f(p') \Gamma_\mu u_f(p)$ where $\bar{u}_f$ and u_f are Dirac spinor fields and Γ_μ has the general Lorentz structure

$$\Gamma_\mu = F_1\left(q^2\right) \gamma_\mu + i F_2\left(q^2\right) \sigma_{\mu\nu} q^\nu - F_3\left(q^2\right) \sigma_{\mu\nu} q^\nu \gamma_5 + F_A\left(q^2\right) \left(\gamma_\mu q^2 - 2m_f q_\mu\right) \gamma_5; \tag{5}$$

with $F_1(0) = Qe$ the electric charge, $F_2(0) = a(Qe/2m)$ the anomalous magnetic moment (anomaly), and $F_3 = dQ$ the electric dipole moment. I ignore the F_A term, the anapole moment.

The anomalous part of the dipole moment interaction

$$\bar{u}_\mu \left[QeF_1(q^2)\gamma_\beta + \frac{iQe}{2m_\mu}F_2(q^2)\sigma_{\beta\delta}q^\delta \right] u_\mu \tag{6}$$

connects states of opposite helicity, i.e. it is chiral changing, giving it a unique sensitivity to New Physics interactions, e.g. the sensitivity to $\tan\beta$ in supersymmetric (SUSY) theories. In most SUSY models, the contribution to a_μ depends on the SUSY mass scale, the sign of the μ parameter, and $\tan\beta$. A simple SUSY model with equal masses [24, 23] gives the SUSY contribution as:

$$\simeq (\text{sgn}\mu)\ 130 \times 10^{-11}\ \tan\beta\ \left(\frac{100\ \text{GeV}}{\tilde{m}} \right)^2 \tag{7}$$

As mentioned above, an EDM violates both P and T symmetries and by implication CP. For hadronic systems, the "theta" term in the QCD Lagrangian[2]

$$\mathcal{L}_{QCD}^{eff} = \mathcal{L}_{QCD} + \theta\frac{g_{QCD}^2}{32\pi^2}F^{a\mu\nu}\tilde{F}_{a\mu\nu}\ \ a = 1, 2, \ldots, 8 \tag{8}$$

violates both parity and time-reversal symmetries, where the physical quantity is the sum of θ and the overall phase in the quark matrix, $\bar{\theta} = \theta + \arg(\det M)$. The non-observation of a neutron EDM restricts the value of $\bar{\theta}$: $|d_n| \simeq 3.6 \times 10^{-16}\bar{\theta}\,e\cdot\text{cm} \Rightarrow \bar{\theta} \leq 10^{-10}$, which for a quantity that could be order one is anomalously small. The smallness of $\bar{\theta}$ is often referred to as *the strong CP problem*. While supersymmetry, or other models of New Physics can easily contain new sources of CP violation, the absence of any observation of an EDM, with a significant fraction of the "natural" part of the SUSY CP-violating parameter space already eliminated, is sometimes called *the SUSY CP problem*.

The isovector and isoscalar combinations of the magnetic dipole moments are:

$$F_{2N}^{(I=1)} = \frac{F_{2p} - F_{2n}}{2} \simeq 1.85\,; \qquad F_{2N}^{(I=0)} = \frac{F_{2p} + F_{2n}}{2} \simeq -0.06\,; \tag{9}$$

we conclude that the isovector dominates the anomalous MDM. Both isoscalar and isovector EDMs are predicted by various models [32], so measuring both the proton and neutron EDMs would help disentangle these two possibilities.

3. Measurements of Dipole Moments
3.1. Measurements of the Muon and Electron Anomalies
The electron anomaly has been measured to a precision of 0.24 parts billion by storing a single electron in a quantum cyclotron and measuring the quantum cyclotron and spin levels in this system [16]. Were an independent measurement of the fine-structure constant α available at this precision, this impressively precise measurement could provide a testing ground for the validity of QED down to the five-loop level, and present an opportunity to search for effects of New Physics. At present the best independent measurements of α have a precision of ~ 5 ppb [17, 18]. In the absence of such an independent measurement, the electron $(g-2)$ value has been used, along with the QED theory (assumed to be valid), to give the most precise value of α [16].

The muon anomaly, which has been measured to an accuracy of 0.54 parts per million (ppm) [19], has an increased sensitivity to heavier physics that scales as $(m_\mu/m_e)^2 \simeq 43,000$.

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[2] In electromagnetic theory, the equivalent sort of term is $\vec{E} \cdot \vec{B}$, which also is odd under P and T. Such a term is important in describing topological insulators in condensed matter physics.

This means that at a measurable level the Standard-Model contributions to the muon anomaly come from QED; from virtual hadrons in vacuum polarization or hadronic light-by-light scattering loops; and from loops involving the electroweak gauge bosons.

In principle the technique is similar to the measurement of the electron anomaly, where muons are stored in a "trap" consisting of a dipole magnetic field plus an electrostatic quadrupole field. In the muon experiment, an ensemble of muons is injected into a precision storage ring. The observable is the spin precession frequency relative to the momentum, which is the difference between the spin precession frequency and the cyclotron frequency:

$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C = -\frac{Qe}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right)\frac{\vec{\beta} \times \vec{E}}{c} \right] . \tag{10}$$

The second term in brackets represents the effect of the motional magnetic field on the spin motion. The experiment is operated at the "magic" value of $\gamma_{magic} = 29.3$ where this motional term vanishes, which permits the use of an electric quadrupole field to provide the vertical focusing.

The measured electron and muon anomalies are

$$a_e = [115\,965\,218\,073(28)] \times 10^{-14}\,0.24\,\text{ppb} \tag{11}$$
$$a_\mu = [116\,592\,089(63)] \times 10^{-11}\,0.54\,\text{ppm} \tag{12}$$

The individual measurements that go into a_μ are shown in Fig. 1(a).

The comparison between the Standard-Model and experimental values of a_μ is partially limited by the knowledge of the hadronic contribution. Significant work on different aspects of the hadronic contribution are in progress, both on the experimental side to measure the hadronic electroproduction cross sections better, and on theoretical efforts to improve on the hadronic light-by-light contribution [20]. There appears to be a difference $\Delta_{a_\mu} = (287 \pm 80) \times 10^{-11}$, 3.6 σ, between the experimental value (Eq. 12) and the Standard-Model value of [21, 22] $a_\mu^{\text{SM}}[e^+e^-] = 116\,591\,802(49)] \times 10^{-11}$.

Such a deviation could fit well with the expectations of supersymmetry in the few-hundred GeV mass region, as shown in Eq. 7. Were SUSY particles to be discovered at LHC, the muon anomaly would play an important role in helping to discriminate between the different possible scenarios, and providing a measure of $\tan\beta$. For a thorough review of SUSY and $(g-2)$ see the articles by Stöckinger [25].

The precision of the E821 $(g-2)$ measurement was limited by the statistical error of 0.46 ppm, compared to the systematic error of 0.28 ppm. A new experiment has been proposed for Fermilab, P989 [27] with the goal of equal statistical and systematic errors, and a total error of 0.14 ppm, a factor of four improvement over E821.

The supersymmetry community has chosen a number of possible scenarios that might be discovered at LHC, the Snowmass points and slopes [26], which serve as benchmarks for determining the sensitivity to the SUSY parameters. Since a_μ has significant sensitivity to $\tan\beta$ (see Eq. 7), it is possible to compare the sensitivity to $\tan\beta$ from LHC vs. from Δa_μ. Such a comparison is shown in Fig. 1(b), which assumes that the SPS1a point is realized, a typical mSUGRA point with an intermediate value of $\tan\beta$. The lighter blue band shows the improvement that could be gained in the new Fermilab experiment.

3.2. Electric Dipole Moments

Unlike the magnetic dipole moments, the Standard-Model values of electric dipole moments are orders of magnitude less than present experimental limits, both of which are shown in Table 1. The the experimental observation of an EDM would unambiguously signify the presence of new physics.
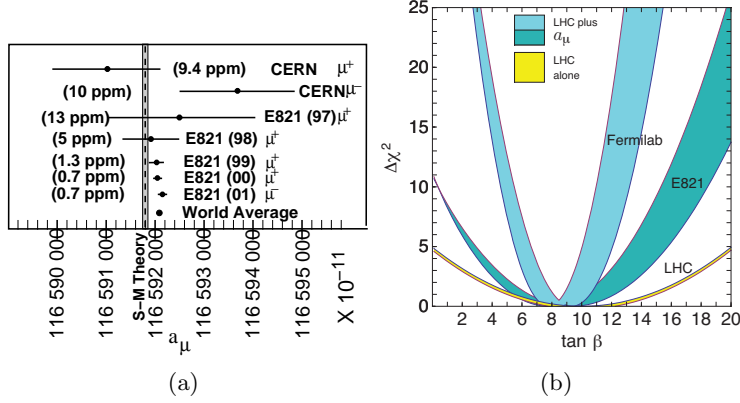
Figure 1. (a) Measurements of the muon anomalous magnetic moment. The theory value shown is taken from Ref. [21] as described in the text. (b) Implications for a $\tan\beta$ determination assuming the LHC were to discover the SPS1a SUSY scenario, which predicts $\Delta a_\mu = 293 \times 10^{-11}$. The a_μ curves assumed $\Delta a_\mu = (255 \pm 80) \times 10^{-11}$, the difference before new data were included in the evaluation of the hadronic contribution [21]. (Figure courtesy of Domink Stöckinger)

Table 1. Measured limits on electric dipole moments, and their Standard Model values

Particle	Present EDM Limit ($e\cdot$ cm)	Standard Model Value ($e\cdot$ cm)
n	2.9×10^{-26} (90%CL) [28]	$\simeq 10^{-32}$
p	7.9×10^{-25} [29]	$\simeq 10^{-32}$
e^-	$\sim 1.6 \times 10^{-27}$ (90%CL) [30]	10^{-38}
μ	$1.8 < 10^{-19}$ (95%CL) [31]	10^{-35}
^{199}Hg	3.1×10^{-29} (95%CL) [29]	

In the traditional EDM experiment, the system is placed in a region of parallel (anti-parallel) electric and magnetic fields (see Fig. 2(a)). The Larmor frequency is measured, and then the electric field direction is flipped. An EDM would cause the Larmor frequency to be higher/lower depending on the direction of the electric field. The EDM is determined by the frequency difference between these two configurations: $\Delta\nu = \nu_{\uparrow\uparrow} - \nu_{\uparrow\downarrow} = (4dE)/(h)$.

The limit on the neutron EDM versus time is shown in Fig. 2(b). The lowest limit for the EDM of any system comes from the ^{199}Hg atom [29]: $d(^{199}\text{H}g) = (0.49 \pm 1.29_{stat} \pm 0.76_{syst}) \times 10^{-29}\, e\cdot$cm giving the limit above in Table 1.

Searches are underway worldwide to find an EDM of the electron [33] (Imperial College, Colorado, Harvard, Yale, Amherst, Penn State, Texas, Osaka and Indiana), neutron [34] (ILL, PSI, Oak Ridge, TRIUMF), the atoms [35] ^{199}Hg (Seattle), ^{129}Xe (Princeton, Michigan), ^{225}Ra (Argonne, Groningen). However, one word of caution, only the neutron, proton, deuteron and muon EDMs can be measured *directly*. All of the other EDM measurements take place in atoms or molecules, and thus their interpretation is subject to issues of atomic and molecular physics.

The limit on the muon EDM comes from E821 at Brookhaven [31]. If an EDM exists, it is necessary to modify the spin precession formula of Eq. 10 with an extra term ω_η,

$$\vec{\omega}_\eta = \eta \frac{Qe}{2m}\left[\frac{\vec{E}}{c} + \vec{\beta} \times \vec{B}\right] \tag{13}$$

and the total spin precession frequency is $\vec{\omega} = \vec{\omega}_a + \vec{\omega}_\eta$. The motional electric field is proportional
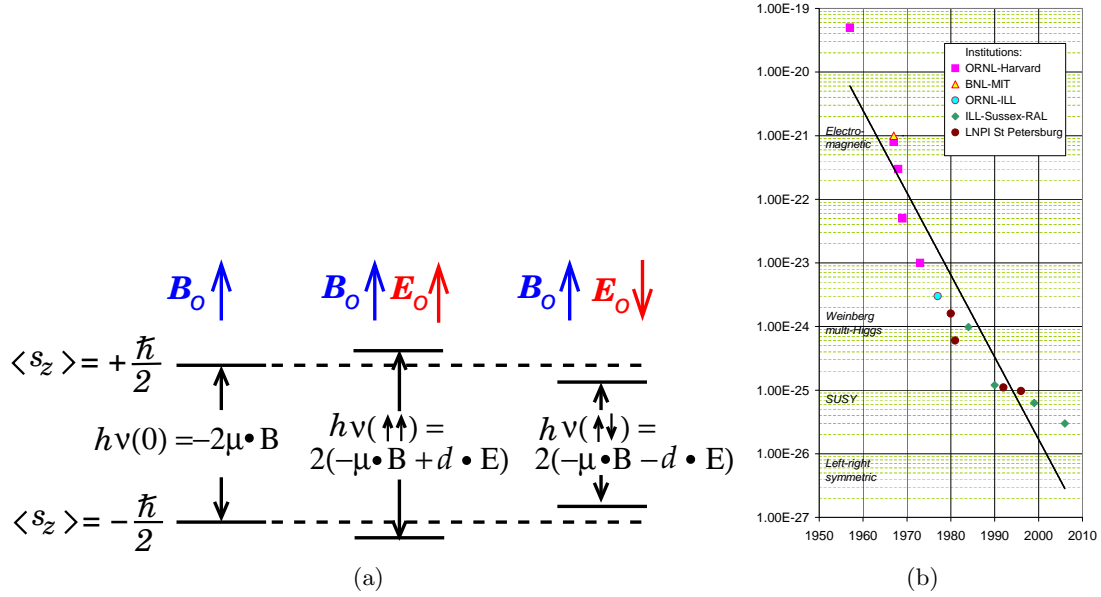
Figure 2. (a) The principle of EDM experiments. (b) Limits on the neutron EDM as a function of time. (courtesy of P.G. Harris).

to $\vec{\beta} \times \vec{B}$, so the EDM results in an out-of-plane component of the spin, where the (very small) tipping angle relative to $\vec{\omega}_a$ is $\delta = \tan^{-1} \omega_\eta/\omega_a = \tan^{-1}(\eta\beta/2a)$. The parameter η is related to the EDM, d, by the relationship

$$d = \left(\frac{e\hbar}{4mc}\right)\eta \text{ for spin } \frac{1}{2}; \text{ and } d = \left(\frac{e\hbar}{2mc}\right)\eta \text{ for spin } 1 . \tag{14}$$

In the $(g-2)$ experiments, $\omega_\eta \ll \omega_a$ and the resulting motion is an up-down oscillation with frequency ω_a, *out of phase* with the $(g-2)$ oscillation. Such an experiment is largely limited by systematic errors [31], since the out-of-plane motion is masked by the large-amplitude spin precession from the magnetic moment. Nevertheless, the new Fermilab $(g-2)$ effort hopes to achieve one to two orders of magnitude improvement in the muon EDM as a by-product of the improved $(g-2)$ measurement. Significant progress beyond that goal would need to reduce the large background caused by the ω_a precession.

To achieve this reduction, the "frozen spin" technique has been proposed [36, 37]. Recall the point of choosing the magic γ in Eq. 10 was to eliminate the effect of the focusing electric field on the spin precession. If however, a storage ring were to be operated at a different momentum, then a *radial* electric field could be used to counter the the spin precession from the magnetic moment (see Eq. 10), *viz.* it could be chosen such that $\omega_a = 0$. The E-field required to freeze the muon spin is [36]

$$E = \frac{aBc\beta\gamma^2}{1 - a\beta^2\gamma^2} \simeq aBc\beta\gamma^2. \tag{15}$$

The frozen spin technique, along with a very high-flux facility, could permit a sensitivity of 10^{-24} $e\cdot$cm or better for the muon EDM, providing a unique opportunity to measure the EDM of a second generation particle.

Both the proton EDM experiment being proposed for Brookhaven, and the deuteron EDM experiment being discussed for COSY present exciting and unique opportunities for direct measurements of hadronic EDMs. Perhaps it is obvious, but should convincing evidence for

any EDM be found, it will be imperative that as many other EDMs as possible be measured to help sort out the source of this new CP violation. More details on measuring EDMs in storage rings are given in the talk by Onderwater, and in Refs. [37, 38].

4. Transition Moments

Although space limitations do not permit a detailed discussion of the searches for lepton flavor violation, I do want to mention briefly the topic of transition moments. One of the most important discoveries in the past decade was the definitive evidence that neutrinos mix. In the Standard Model, this implies that charged leptons will also mix, however the calculated transition rate for $\mu^+ \to e^+\gamma$ is: $Br(\mu \to e\gamma) = (3\alpha)/(32\pi)\left|\sum_\ell V^*_{\mu\ell}V_{e\ell}(m^2_{\nu_\ell})/(M^2_W)\right|^2 \leq 10^{-54}$, which is immeasurable under the most optimistic experimental scenario. Thus the observation of any process that violates lepton flavor, such as $\mu^+ \to e^+\gamma$, $\tau^+ \to \mu^+\gamma$, or coherent muon to electron conversion, $\mu^- + \mathcal{N} \to e^- + \mathcal{N}$, would herald the discovery of new physics.

Just as the diagonal matrix elements of the electromagnetic current were connected with the electric and magnetic dipole moments, we have the off-diagonal elements of the current [24] that give transition moments: $\left\langle f_j\left(p'\right)\left|J^{\mathrm{em}}_\mu\right|f_i\left(p\right)\right\rangle = \bar{u}_j\left(p'\right)\Gamma^{ij}_\mu u_i\left(p\right)$, where Γ^{ij}_μ is given by

$$\Gamma^{ij}_\mu = \left(q^2 g_{\mu\nu} - q_\mu q_\nu\right)\gamma^\nu\left[F^{ij}_{E0}\left(q^2\right) + \gamma_5 F^{ij}_{M0}\left(q^2\right)\right] + i\sigma_{\mu\nu}q^\nu\left[F^{ij}_{M1}\left(q^2\right) + \gamma_5 F^{ij}_{E1}\left(q^2\right)\right]. \quad (16)$$

The first term gives rise to chiral-conserving flavor-changing amplitudes at $q^2 \neq 0$, e.g. $K^+ \to \pi^+ e^+ e^-$, $\mu^+ \to e^+ e^+ e^-$, and the second (dipole) term gives rise to chiral-changing, flavor-changing amplitudes, e.g. $b \to s\gamma$, $\mu \to e\gamma$ and $\tau \to e\gamma$. The search for these Standard-Model forbidden decays provides a complementary path to discover new physics, and in some models the muon anomaly, EDMs and charged lepton flavor violation are connected. For more details, see the reviews of charged lepton flavor violation [39, 40, 41, 42].

5. Summary and Conclusions

Spin physics began with the paper of Uhlenbeck and Goudsmit [2] which explicitly introduced the concept of a magnetic moment associated with electron spin. Twenty five years later, Purcell and Ramsey [12] proposed to search for an electric dipole moment to look for New Physics (parity violation). We now recognize Dipole moments as an essential tool in the search for physics beyond the Standard Model.

There may already be an indication of New Physics at the loop level from the muon $(g-2)$ experiment. The new experiment to measure a_μ a factor of four more precisely at Fermilab should clarify the difference that has been observed between the Standard-Model value and experiment. The observation of an electric dipole moment would herald the discovery of a new source of CP violation, which we believe must exist to explain the matter-antimatter asymmetry of the universe. Similarly, a discovery in the on-going searches for charged lepton flavor violation at the Paul Scherrer Institut, Fermilab, J-PARC, and the B Factories would also herald New Physics at work in the charged lepton sector.

All of these experiments will help to guide our interpretation of the new phenomena which we hope to discover at LHC. There are a number of opportunities to make important contributions to this field that are open to the spin physics community. Of special note are the opportunities being discussed for Brookhaven and for the COSY facility in Jülich to measure *directly* the proton and deuteron EDMs using the storage-ring technique.

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